

Explicit K -moduli seminar

- (1) Overview, including several important criterion of K -stability: δ -invariant, β -invariant ... and giving some concrete questions.
(Si)
- (2) ADL's paper: general constructions (4 weeks)
 - (a) Boundedness : Wu
 - (b) separatedness: Zhang
 - (c) projectivity: Kollar's ampleness lemma.
 - (d) CM line bundle: Si
 - (e) Wall-crossing: Pan
- (3) Explicit examples:
 - (a) Liu-Xu and Liu's cubic 3 and 4 fold example (Si) (two weeks)
 - (b) ADL's compactification of low genus K3 (two weeks)

Problems:

- (1) Generalise [OSS] to the log setting, ie, study the K -moduli $M_{d,\epsilon}^{k-ps}$ of K -polystable log Fano surface smoothing by $(S, \epsilon C)$ where S is a del pezzo surfaces of degree $d = (-K_S)^2 \in \{1, \dots, 9\}$ and $C \in |-K_S|$, $\epsilon \in \mathbb{Q}$.
It is more interesting to compare the K -moduli compactification to the metric compactification $M_{d,\epsilon}^{GH}$ via Gromov-Hausdorff metric.
Note recently, Yu-Keel study moduli of log CY surfaces via tropical geometry and Non-archimedean geometry, maybe the wall-crossing from varying ϵ will relate to their work ?
- (2) Related to moduli of degree 8 K3 : explicitly construct the moduli M_t^{k-ps} of log K -polystable 3 fold pair smoothing by $(X, t\Delta)$ where $X \subset \mathbb{P}^5$ is a complete intersection of two quadrics and $\Delta \in |-K_X|$, $t \in [0, 1) \cap \mathbb{Q}$. It should be similar to degree 6 case (this case can be related to the recent work of VGIT). More challenging problem is to allow $t \geq 1$, then the moduli objects are log CY or log general type. the wall-crossing phenomenon is unknown.
- (3) study the K -moduli M_d^{K-ps} of prime Fano 3-fold of degree $d = (-K_X)^3$. eg, $d = 2$, related to moduli of K -ps pair smoothing by (\mathbb{P}^3, S) where $S \subset \mathbb{P}^3$ sextic hypersurfaces; $d = 22$ is also interesting.
- (4) K -moduli GM_n^{k-ps} of Gushel-Mukai n -folds ($3 \leq n \leq 5$). In these cases, the K -stability may be very hard to check.

These days, Gushel-Mukai n -folds have attracted lots of attentions, eg, its Derived categories, its rationality and its period map (see Debarre-Kuznestov's work).

- (5) Dervan-Ross generalise K-stability to a morphism $X \rightarrow Y$. It will be interesting to use the generalised K-stability to construct the moduli with modular meaning.